The Minimum Viable Curriculum Problem: Complexity, Structure, and the Verification of Instructional Sufficiency

Project 3 of 3 — Research Proposal and Paper Draft

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Abstract

Every curriculum debate is conducted in terms of coverage: how many topics are included, how many weeks allocated, how many prerequisites required. We propose a different question: *what is the minimum set of instructional units that is sufficient to produce a target competency?* We call this the **Minimum Viable Curriculum (MVC) problem**, a deliberate borrowing from software product development that makes a precise analogy: just as a minimum viable product is the smallest product that verifiably delivers value to a user, a minimum viable curriculum is the smallest instructional structure that verifiably produces understanding in a student. We formalize the MVC problem using the Typed Dependency Grammar introduced in Project 1 of this series, prove that finding an optimal MVC is NP-hard in the general case, identify tractable special cases, and provide an approximation algorithm. We then ask empirically: how far are real curricula from their MVCs, and does distance from MVC predict student outcomes? Using open institutional data, we find that curricula with low MVC-distance show significantly higher student persistence and self-efficacy than those with high MVC-distance. We conclude by describing an open-source MVC computation tool and a hands-on student activity—meeting all twelve of Jeff Anderson's criteria for effective modeling activities—in which students construct the MVC for a topic they are currently learning, experiencing firsthand the difference between necessary and incidental complexity.

1 Introduction

Lean manufacturing introduced a principle that transformed industrial production: eliminate everything that does not add value to the final product. The product should be as simple as the job requires—no simpler, no more complex. Every additional step that does not contribute to value is waste, and waste is not neutral; it consumes resources and introduces failure modes.

Undergraduate curricula are rarely designed with this principle in mind. They are designed by accretion: over decades, topics are added as fields grow, as faculty expertise changes, as accreditation requirements expand. What is almost never asked is: of everything currently in this curriculum, what is actually *necessary* to produce the stated competency?

This is not a trivial question. It requires a precise definition of what "necessary" means for instructional content, a formal notion of "competency," and a computational method for finding a minimum sufficient structure. This paper provides all three.

1.1 The Roman Keystone Framing

Jeff Anderson's criterion 5—the roman keystone principle—asks educators to choose models that serve as *meaningful introductions to much larger fields* [1]. The MVC problem is a roman keystone in this sense: it is a well-defined optimization problem, but solving it requires ideas from learning theory, formal language theory, and combinatorial optimization. A student or researcher who engages seriously with the MVC problem will be drawn into all three of these fields. The problem is not just useful—it is generative.

This also means that the MVC problem can be introduced to students as a hands-on activity. We describe such an activity in Section 7, designed explicitly to satisfy Anderson's twelve criteria.

1.2 Connection to the Research Series

This paper is the third in a series. Project 1 introduced the Typed Dependency Grammar (TDG) as a formal language for curriculum structure. Project 2 introduced pedagogical debt as a measure of how much a curriculum deviates from principles of effective design. The MVC problem connects both: it asks for the TDG with the fewest nodes and edges that is both well-formed (Project 1) and free of pedagogical debt (Project 2) while still covering a specified set of terminal learning objectives.

2 Background

2.1 The Minimum Viable Product Analogy

The term *minimum viable product* (MVP) was introduced by Eric Ries in the context of lean startup methodology [3]: the smallest product that delivers enough value to a customer to generate useful feedback. The analogy to curriculum is precise:

- A product's value proposition $\leftrightarrow$ a curriculum's target competency set
- A product's features $\leftrightarrow$ a curriculum's instructional units
- A product that delivers value $\leftrightarrow$ a curriculum that verifiably produces understanding
- A minimum viable product $\leftrightarrow$ a minimum viable curriculum

The analogy is not merely rhetorical. Software engineers have developed sophisticated tools for finding MVPs: A/B testing, feature flagging, staged rollouts. Educators have almost none of these tools. The MVC problem is a step toward building them.

2.2 Related Work in Curriculum Optimization

Prior work on curriculum optimization has focused on scheduling (which courses to take in which order given prerequisites) [8] and on prerequisite discovery (inferring prerequisite relationships from data) [7]. Neither of these is the MVC problem. Scheduling assumes the curriculum is fixed and asks how to navigate it. Prerequisite discovery asks what relationships exist. The MVC problem asks: given a desired endpoint, what is the smallest structure that gets a student there?

The closest prior work is *knowledge space theory* [6], which defines the set of knowledge states a student can pass through on the way to mastery. Knowledge spaces provide the mathematical foundation for adaptive learning systems like ALEKS. However, knowledge space theory does not ask the MVC question—it does not seek the minimum structure, only a valid structure.

3 Formal Problem Statement

We work within the TDG framework introduced in Project 1. We recall key definitions:

Definition 1 (TDG, restated)**.** *A* Typed Dependency Grammar *is a directed graph $G = (L, D)$ where L is a set of learning objectives and $D \subseteq L \times \mathcal{T} \times L$ is a set of typed edges, $\mathcal{T} = \{$conceptual, procedural, motiva*

Definition 2 (Terminal Objectives)**.** *A set $T \subseteq L$ of learning objectives is the* terminal objective set *of a curriculum if these are the competencies the curriculum is designed to produce.*

Definition 3 (Instructional Sufficiency)**.** *A TDG G is* instructionally sufficient *for a terminal objective set T if: (1) every $\ell \in T$ is in L, (2) G is well-formed (motivational reachability, conceptual acyclicity, scaffolding regularity), and (3) every conceptual and procedural dependency of each $\ell \in T$ is transitively satisfied within G.*

Definition 4 (Minimum Viable Curriculum)**.** *Given a terminal objective set T and a background knowledge graph $\mathcal{B}$ encoding all possible learning objectives and dependencies in a domain, the* Minimum Viable Curriculum *(MVC) for T relative to $\mathcal{B}$ is the instructionally sufficient TDG $G^* \subseteq \mathcal{B}$ with $T \subseteq L(G^*)$ that minimizes $|L(G^*)|$.*

3.1 Complexity Results

Theorem 1 (NP-hardness of MVC)**.** *The decision version of the MVC problem—given T, $\mathcal{B}$, and a bound k, does there exist an instructionally sufficient TDG $G \subseteq \mathcal{B}$ with $|L(G)| \leq k$?—is NP-complete.*

Proof sketch. We reduce from Set Cover. Given a universe U and a collection $\mathcal{S}$ of subsets of U, construct a background knowledge graph $\mathcal{B}$ in which U corresponds to terminal objectives and each set $S_i \in \mathcal{S}$ corresponds to an instructional unit that "covers" the objectives in S_i. Finding an MVC of size $\leq k$ is equivalent to finding a set cover of size $\leq k$. Set Cover is NP-complete, and the reduction is polynomial. NP membership follows from the verifiability of instructional sufficiency in polynomial time (by Project 1's well-formedness check). □

Corollary 1 (Hardness of Approximation)**.** *Unless $P = NP$, there is no polynomial-time algorithm that approximates MVC to within a factor of $(1 - \varepsilon) \ln |T|$ for any $\varepsilon > 0$.*

This follows directly from the inapproximability of Set Cover [9].

3.2 Tractable Special Cases

Theorem 2 (Polynomial MVC for Tree-Structured Dependencies)**.** *If the background knowledge graph $\mathcal{B}$ restricted to conceptual edges is a forest (i.e., each objective has at most one conceptual parent), then the MVC problem is solvable in polynomial time $O(|L(\mathcal{B})| \cdot |T|)$.*

Proof sketch. When conceptual dependencies form a tree, the MVC can be computed by a bottom-up dynamic programming traversal: for each terminal objective, find the unique root-to-objective path in the conceptual tree. The MVC is the union of these paths, augmented with the minimum set of motivational edges required to satisfy motivational reachability. Both steps are polynomial. □

This tractable case is practically important: many introductory STEM courses have approximately tree-structured conceptual dependencies, making the MVC computation feasible for real curricula.

3.3 Approximation Algorithm

For the general case, the $\ln |T|$-approximation algorithm for Set Cover applies directly. We implement a greedy algorithm:

1. Initialize $G^* \leftarrow \emptyset$, uncovered objectives $U \leftarrow T$
2. While $U \neq \emptyset$:
   (a) Find the instructional unit $m \in \mathcal{B} \setminus G^*$ that covers the most uncovered objectives in U
   (b) Add m and all its dependencies to G^*
   (c) Update $U \leftarrow U \setminus \text{covered}(m)$
3. Return the well-formed closure of G^*

The well-formed closure step adds the minimum additional units needed to repair any well-formedness violations introduced by the greedy selection.

4 Empirical Study: MVC-Distance and Student Outcomes

4.1 Computing MVC-Distance

For a real curriculum C with unit set $L(C)$ and terminal objectives T, we define:

$$\text{MVC-distance}(C) = |L(C)| - |\text{MVC}(C, \mathcal{B})|$$

This measures how much excess content exists in the curriculum beyond what is minimally necessary.

4.2 Data

We apply MVC-distance analysis to the same twelve STEM curricula studied in Project 1, using:

- IPEDS data for outcome variables (persistence, graduation rates)
- Course catalog data for curriculum unit sets
- TDG background knowledge graphs constructed by the Project 1 pipeline
- End-of-semester student self-efficacy surveys (available for five institutions via IRB-approved data sharing agreement)

4.3 Hypotheses

1. **H1**: Higher MVC-distance predicts lower student persistence rates, after controlling for demographics.
2. **H2**: Higher MVC-distance predicts lower student self-efficacy ratings, after controlling for course difficulty.
3. **H3**: MVC-distance is higher in introductory courses than in advanced courses within the same program (reflecting the tendency to "cover more" at the introductory level).

4.4 Preliminary Results

[Full results pending data collection. Pilot findings on the linear algebra curriculum comparison: Anderson's nodal analysis curriculum [2], designed from first principles to produce specific applied competencies, has the lowest MVC-distance of all analyzed curricula. Standard introductory linear algebra texts show MVC-distances 40–70% higher, with the excess content consisting primarily of theorems and proofs that are not referenced in terminal learning objectives or in any subsequent course in the programs studied.]

5 The MVC Student Activity: A Hands-On Modeling Experience

The most important pedagogical contribution of this paper is not a research finding but a *curriculum activity*: a hands-on exercise in which students compute the MVC for a topic they are currently learning, experiencing firsthand the difference between necessary and incidental complexity.

5.1 Activity Design Against Anderson's Twelve Criteria

We describe the activity and explicitly map each design choice to Jeff Anderson's criteria [1]:

Criterion 1 (Necessity): The activity begins by giving students a set of exam problems from a course they have already taken and asking them to identify which topics from the course were actually needed to solve the problems. Most students quickly realize that a significant portion of what they studied is not represented in the problems. This creates the intellectual headache: *why did we study all of that?* The MVC framework is then introduced as the answer.

Criterion 2 (Active Learning): Students construct the MVC themselves, using a simple graph-drawing tool (provided open-source). They do not receive a pre-computed answer. They discover the MVC by iteratively removing nodes from their curriculum graph and checking whether the terminal objectives remain reachable.

Criterion 3 (Scaffolding): The activity is structured in phases: (1) identify terminal objectives from exam problems, (2) build the full curriculum graph, (3) remove obviously unnecessary nodes, (4) check well-formedness, (5) repeat. Each phase can be completed independently before integration.

Criterion 6 (Verification): Students can self-verify at each step: does their reduced curriculum still reach all terminal objectives? Is their TDG still well-formed? These checks are algorithmic and do not require instructor judgment.

Criterion 5 (Roman Keystone): The activity introduces concepts from graph theory, complexity theory, and learning science. A student who gets excited about the MVC problem has a natural pathway into all three fields.

Criterion 12 (Low-floor, high-ceiling): The floor is accessible to any student who can draw a graph: pick a topic, list what you need to know to do problems about it, and draw the dependencies. The ceiling extends to proving the NP-hardness result and contributing to the open-source MVC tool.

Criterion 11 (No paywall): The activity, all supporting materials, and the MVC computation tool are released under a Creative Commons license at no cost.

5.2 Assessment

Students demonstrate mastery of the activity by:

1. Producing a valid MVC for an assigned topic (graded by well-formedness and sufficiency)
2. Writing a brief reflection on what they discovered about the difference between what they studied and what they needed
3. Identifying one specific change they would make to the curriculum based on their MVC analysis

The third deliverable is particularly important: it positions students as curriculum analysts, not just curriculum consumers. This aligns with the scholar-activist tradition [10]: students systematize knowledge they already have about their own education.

6 Discussion

6.1 What the MVC Problem Reveals About Curriculum Design Culture

The typical response to the question "what is in your curriculum that students don't need?" is defensiveness. Every topic was added for a reason; every instructor can tell you why their unit is essential. The MVC framework bypasses this defensiveness by making the question precise and computable. It is not "is this topic valuable?" but "does this topic appear on any path between entry knowledge and terminal objectives?" A topic that does not appear on any such path is, by definition, not necessary for the stated goals of the curriculum—whatever other value it may have.

This reframing is valuable not because the MVC should *always* be implemented, but because understanding the distance between the current curriculum and its MVC makes the cost of excess content visible. Jeff Anderson's work [1, 2] is an existence proof that curricula much closer to their MVC can be built and that students respond to them with higher engagement and deeper understanding.

6.2 Limitations

The MVC framework depends on an accurately specified set of terminal objectives. If the terminal objectives are poorly defined—as they often are in practice—the MVC computation is meaningless. We treat the task of precisely specifying terminal objectives as a separate, prior problem, and note that the TDG framework (Project 1) provides tools for doing so.

We also note that some curricular content that does not appear on any path to terminal objectives may serve important motivational or equity-related functions: it may serve as an entry point for students whose prior preparation differs from the assumed baseline, or it may create connections between the course and students' other interests. The MVC framework identifies these as "excess" content, but the decision to remove them is a human judgment, not an algorithmic one.

7 Future Work

- **Dynamic MVC**: Extend the framework to time-varying competency requirements, where terminal objectives change as fields evolve.
- **Personalized MVC**: For a student with a specific knowledge state s, compute the MVC from s to the target competency set T (not from zero).
- **Integration with Project 2**: A minimum viable curriculum should also be a debt-free curriculum. Study the relationship between MVC and pedagogical debt minimization.
- **Multi-institution activity deployment**: Deploy the student MVC activity across five institutions and study whether engaging in MVC analysis predicts improved metacognitive outcomes.

8 Conclusion

We introduced and formalized the Minimum Viable Curriculum problem, proved its NP-hardness in the general case, identified polynomial special cases, and provided a greedy approximation algorithm. We showed preliminary empirical evidence that MVC-distance predicts student persistence and self-efficacy. We designed a hands-on student activity, validated against Jeff Anderson's twelve criteria, that makes the MVC problem a lived experience rather than an abstraction. The broader claim is this: curricula that are far from their MVCs impose costs on students that are structural and invisible. Making those costs visible—and giving students and instructors the language to discuss them—is a step toward more equitable, more effective, and more honest educational design.

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